

## Enrichment 5-8

### *Coefficients of Complex Quadratic Equations*

Recall that a complex number has the form  $x + yi$ , where both  $x$  and  $y$  are real numbers. If  $x = 0$ , the number is a pure imaginary number; if  $y = 0$ , the number is a real number.

Suppose that  $ax^2 + bx + c = 0$  is a quadratic equation whose coefficients  $a$ ,  $b$ , and  $c$  are complex numbers. Such an equation can still be solved using the standard quadratic formula. However, keep in mind that if the coefficients are complex numbers, the complex roots need not be conjugates. Other properties hold as well. The sum of the roots is  $-\frac{b}{a}$ , and the product of the roots is  $\frac{c}{a}$ .

Let  $ax^2 + bx + c = 0$  be a quadratic equation with nonzero roots. For each problem that follows, some information about the roots of the polynomial is given. Determine whether each statement is true or false. If false, give a counterexample. A monic polynomial is a polynomial in which the coefficient of the term of highest degree is 1.

1. The polynomial is monic, and both roots are nonzero pure imaginary.
  - a.  $c$  must be real
  - b.  $b$  must be real
  - c.  $b + c$  cannot be 0
  - d.  $bc$  cannot be 0
2. The coefficient  $a$  is real, and the sum of the roots is real.
  - a.  $b$  must be real
  - b.  $c$  must be real
  - c.  $b + c$  cannot be 0
  - d.  $bc$  cannot be 0
3. The polynomial is monic, and the product of the roots is real.
  - a.  $b$  cannot be real
  - b.  $c$  cannot be real
  - c.  $b + c$  cannot be real
  - d.  $c$  cannot be 0